The interplay of sampling and accuracy in gravity surveys

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Summary

When presented with a gravity data set for any area, two numbers are typically of interest: the resolution and the accuracy. The resolution is the spatial size of the smallest features visible in the data; the accuracy quantifies the reliability of the features in the data. It is well understood that to resolve features, an area must be sampled with a spacing that is small enough to see those features. It may be less obvious that sampling can play an important role in determining the accuracy of a survey, either through undersampling or oversampling. The different sampling patterns of ground or airborne gravity surveys affect the accuracy of the resultant grids.

Introduction

Nyquist–Shannon sampling theorem states: “If a function f(t) contains no frequencies higher than W [counts per second], it is completely determined by giving its ordinates at a series of points spaced 1/2 W seconds apart” (Shannon, 1949). If the sampling is inadequate and done at an interval more than 1/2 the wavelength, the signal may be aliased (Figure 1).

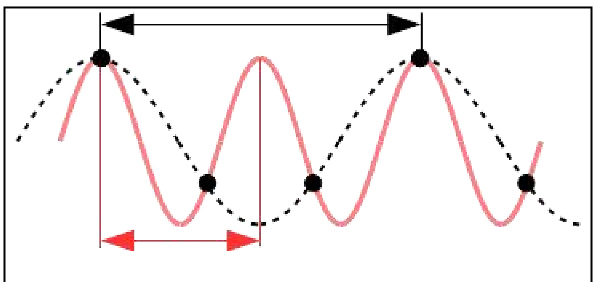

Figure 1: The black signal is completely determined (sampling < 1/2 wavelength). A shorter wavelength signal in red is aliased (sampling > 1/2 wavelength) and is indistinguishable from the longer wavelength signal.

Discussions about sampling theorem, and examples like the image above, tend to focus on the aliasing of a repeating time-based signal. In gravity surveys we are sampling a non-repeating signal, the gravity field, in space. In this context, the sampling theorem often gets shortened to a simple rule of thumb: in order to resolve a feature, one should sample with a spacing that is 1/2 the size of the feature. But the theorem only applies *if there are no higher frequencies present*. This is usually not the case. The presence of shorter wavelengths (i.e., higher frequencies) means wavelengths of twice the sample spacing cannot be completely determined. A survey may represent the gravity in an area inaccurately simply through how the area is sampled.

This should be kept in mind when considering the results from a gravity survey. Accuracy is not determined solely by the instrument. Sampling plays a role in accuracy too. From a geophysical perspective, even a perfectly accurate measurement point can be an imperfect representation of the surrounding area due to the near-station effects of inhomogeneity.

Sampling at points:

The impact of sampling on accuracy can be illustrated with high-resolution data that is re-sampled using subsets of points to simulate lower resolution surveys. The original tightly spaced higher resolution data serves as a reference gravity field. The R. J. Smith airborne gravity test range at Kauring, Western Australia (the Kauring test range), is convenient for this purpose. It contains publicly available high resolution ground gravity in an area that has been specifically designed for comparisons with and between airborne gravity systems (Howard, 2010, and Daishsat Geodetic Surveyors, 2009). A free air gravity grid from the Kauring test range is displayed below in Figure 2 using the original 500 m ground station spacing.

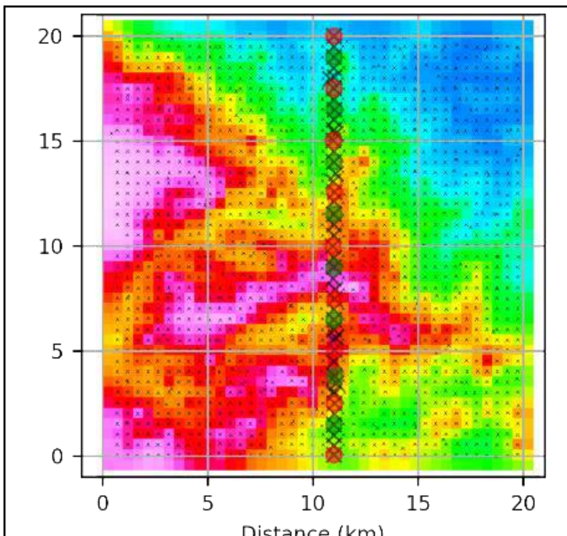

Figure 2: Free air gravity, 500 m point spacing. Central N-S line is used for profiles in Figure 4.

Also shown are a 5000 m full-wavelength filtered version of the grid, representing a ‘regional’ gravity field, and the

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difference between the filtered and original grid (Figure 3). Obviously there are shorter wavelengths than 5000 m present.

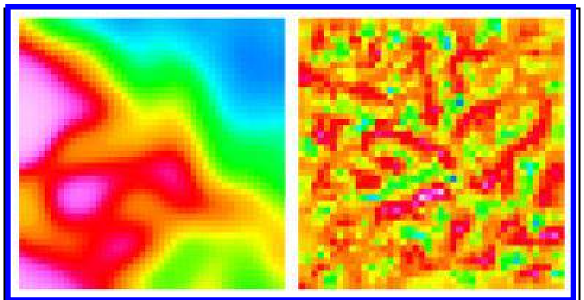

Figure 3: 5000 m full-wavelength filtered grid (left); biasing shorter wavelengths (right).

A single North-South oriented profile near the centre of the area has been selected to highlight what happens when a ground track is sampled using 2500 m spaced points with the intent of creating a 5000 m resolution regional gravity profile (Figure 4). The two 2500 m spaced profiles are distinctly different where they encounter short-wavelength anomalies between the 5 and 10 km mark, since they are sampling at different locations. The smoothed 'regional' gravity (sampled along the same track from the left grid in Figure 3) is included for comparison.

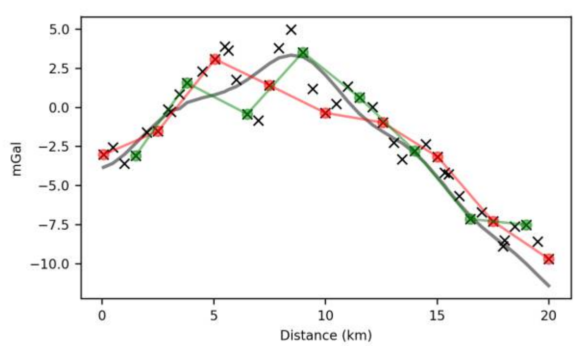

Figure 4: Original 500 m points (x's) with 2500 m sub-sampled profiles (red and green lines). 'Regional' gravity is the smoother black line.

Moving from 1-D to 2-D, a similar re-sampling procedure can be performed for the entire Kauring test range (Elieff, 2017). The original 500 m spaced measurements have been subdivided into 5 sets of 2500 m spaced measurements, offset from each other by 500 m in both the X and Y directions. This is analogous to performing 5 independent regional gravity surveys in the same area. The results are gridded and filtered with identical 5000 m full-wavelength grid filters (Figure 5). While the longer wavelengths agree well in these grids, there are differences. The shorter wavelengths present in the area (right panel of Figure 3) are biasing these grids.

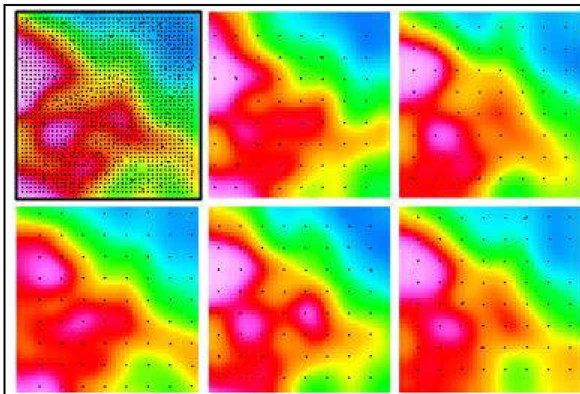

Figure 5: Original 500 m spaced survey (top left); 5 subsets using 2500 m spacing, offset from each other (other panels). A 5000 m full-wavelength filter has been applied.

Sampling along lines:

So far in these examples we have assumed that individual ground measurements have been made perfectly, contributing no error. What if we extend that assumption and imagine we have a perfect airborne gravity meter, flown by expert pilots at ground level, along 'flight' lines? We can approximate this by dividing the Kauring test range ground points into 5 sets of 2500 m spaced lines, each offset by 500 m in the X direction. The data is then gridded and filtered as before (Figure 6).

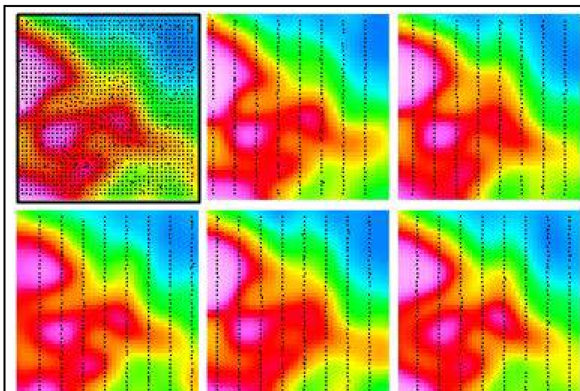

Figure 6: Original 500 m spaced survey (top left); 5 subsets sampling along 2500 m spaced N-S lines, offsets from each other (other panels). A 5000 m full-wavelength filter has been applied.

This shows an improvement over the 2500 m spaced points in the previous example. Table 1 gives the standard deviations of the differences between sub-sampled grids and the original grid in Figure 5 and Figure 6 for both free air (shown above) and Bouguer gravity (not shown).

	2500 m spaced points (simulating ground survey)	**2500 m spaced points (simulating airborne survey)**
Free air	1.2 mGal	0.7 mGal
Bouguer	0.5 mGal	0.3 mGal

Table 1: Standard deviation of the differences between sub-sampled grids and the original grid.

Two things are apparent. First, all things being equal, simply sampling an area more completely using 2500 m spaced ‘flight’ lines will give a better result than sampling with 2500 m spaced stations. Second, the degree to which there is an improvement depends on how much higher frequency signal there is. If there are fewer higher frequencies, there are fewer potential aliasing issues. By applying a Bouguer correction to remove terrain effects, which removes higher frequency signals in this data set, the agreement between the original fully sampled grid and the sub-sampled grids improves. The sections of the sub-sampled profiles in Figure 4, past the 10 km mark, are in better agreement than the sections in the 5 to 10 km mark for the same reason.

Obviously all things are not equal in actual ground and airborne gravity surveys. The error level of a single ground reading is smaller than the error of a single airborne gravity flight line. Ultimately, however, what matters most is how well an entire data set represents the gravity field in an area, not how accurate an individual measurement point or flight line is.

This distinction is an important one. In 2016, the Geological Survey of Western Australia and Geoscience Australia began using airborne gravity surveys to continue a program of reconnaissance gravity mapping that, up to that point, had been done using ground gravity. A major concern was ensuring both the spatial resolution and accuracy of an airborne gravity survey would be comparable to ground data with 2500 m station spacing. A survey was flown with 2500 m flight line spacing using a Sander Geophysics AIRGrav system. Results showed the airborne gravity survey results were comparable to ground gravity surveys (Howard, 2018).

This can be understood using the Bouguer gravity results from Table 1 in a simplified error calculation. Individual AIRGrav 5000 m full-wavelength lines had an estimated 0.5 mGal accuracy in Howard (2018). Adding the 0.3 mGal sampling accuracy for a line (bottom right, Table 1), the total error is $\sqrt{(0.5 + 0.3)} = 0.6$ mGal. This is very similar to the 0.5 mGal accuracy estimated from the grids created from 2500 m station spacing (bottom left, Table 1). While this is an approximation made using some assumptions (the survey area in Howard (2018) is assumed to have similar gravity signal frequency content to the Kauring test range, the ground gravity measurements have been assigned no error, actual airborne measurements are made at a much higher rate than implied by 500 m spacing), it demonstrates why the overall results for the airborne survey were comparable to ground surveys despite differences in the accuracy of individual measurements.

Oversampling:

While undersampling may introduce errors, oversampling can reduce them. If measurements contain random error, repeated sampling allows averaging of results and error reduction. In a gravity survey, oversampling an area with tight line spacing relative to the size of a spatial (grid based) filter allows averaging across adjacent lines. This removes uncorrelated signals, producing a more accurate final survey result (Sander, 2003). This is analogous to signal stacking in seismic surveys, or flying the same line back and forth repeatedly to create an average profile with higher accuracy. The oversampling process is illustrated in Figure 7 .

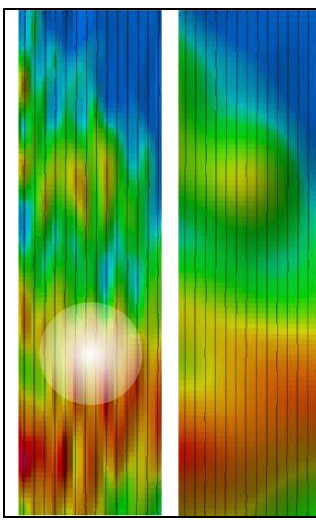

Figure 7: An area is oversampled with close line spacing and gridded (left). The grid based spatial filter is represented by the white circle. In addition to smoothing along the lines, the spatial filter averages across the lines, removing uncorrelated random noise. The resulting spatially filtered grid (right) will have a lower noise level than the individual lines that produced it

The Kauring test range again provides an example. The range was flown using an AIRGrav system in 2012 with 50 m line spacing in a central 5x5 km area and 200 m line spacing in the outer 20x20 km area (Sander Geophysics, 2012). Spatial filters of 600 m and 1000 m half-wavelength were used on gridded line data in each area respectively. Since these filters are larger than the 200 m line spacing, the area is oversampled relative to the filters

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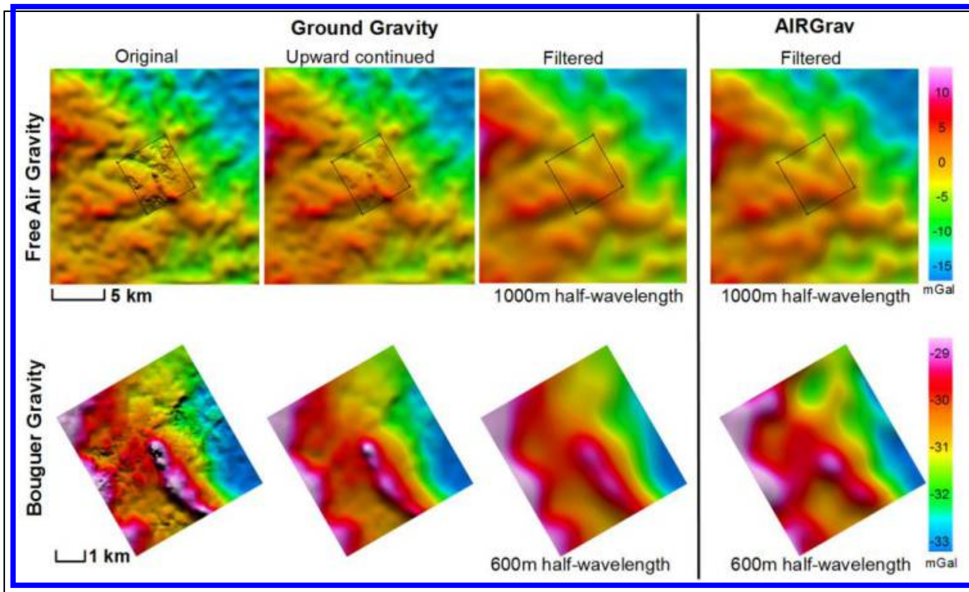

Figure 8: Comparison of ground and AIRGrav airborne gravity grids from the Kauring test range. The 20x20 km area (top row) was flown using 200 m line spacing; the inner 5x5 km area (bottom row) using 50 m line spacing. Relative to the grid filters applied, these spacings are oversampling each area. The ground gravity is upward continued and filtered to match the airborne data for the comparison, as shown.

which determine the resolution. The accuracy of the airborne data was estimated by a comparison with the ground data in the area (Figure 8). A significant improvement in resolution and accuracy is gained relative to the resolution and accuracy of a single flight line (Figure 9).

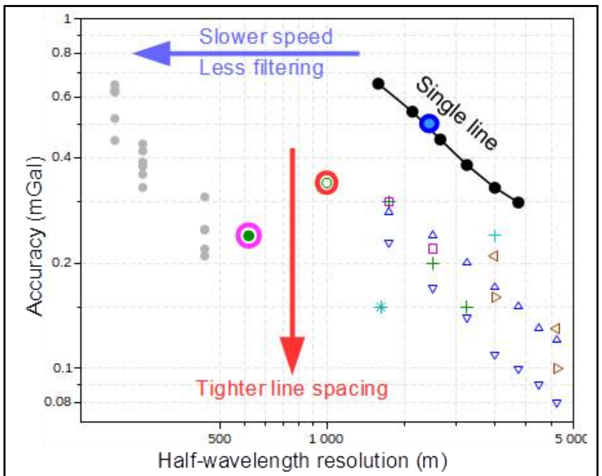

Figure 9: AIRGrav accuracy and resolution of a single line flown with a fixed wing aircraft (black line/points), and final overall results of grid filtering multiple lines from a wide range of surveys (other points). Kauring test range results at 600 m and 1000 m resolution are circled (pink and red), as well as the estimated accuracy for a 2500 m half-wavelength filtered single line from the East Kimberley survey (blue).

The broad range of resolutions and accuracies from many different projects (also shown in Figure 9) illustrates the interplay between sampling, accuracy, and resolution. The results shown are from the AIRGrav system, where the primary source of error is random time-based GPS noise. Since GPS noise is time based, flying slower will improve resolution. Since the noise is random, oversampling with tighter line spacing will improve accuracy. The range of survey results expands far beyond that of a single line.

Conclusions:

Sampling can play a significant role in determining the accuracy of a survey. A collection of individual ground readings or flight lines in a database is not the final product of a survey. The readings are generally converted into a representation of the continuous gravity field, such as a profile or a grid. This involves making assumptions about the gravity field in places where it was not sampled, which can introduce errors. If the points or lines in a database have oversampled an area, the representation of the gravity field derived from a survey will be improved beyond that of the individual measurements.

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